

The Norwegian vertical reference frame NN2000

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Preface

We started writing this report in 2008 when the Norwegian leveling network was first calculated. Originally, the intention was to cover just the realization of NN2000 in the leveling network. When the final calculation of the leveling network was ready in 2012, further writing stopped. When we years later resumed writing, the calculation of NN2000 in the GNSS network and the implementation in the municipalities were almost completed. It was natural to include also a documentation of these tasks in this report.

We hope this report will serve as a documentation on how we realized NN2000 in Norway, both theoretically and in practice. In addition to surveyors, geodesists, geophysicists, and cartographers, we think foreign companies operating in Norway may be potential readers. For this last group we have written the report in English.

Chapter [1](#page-5-0) to [5](#page-25-0) cover the realization of NN2000 in the leveling network, Chapter [6](#page-38-0) the realization in the GNSS network, and Chapter [7](#page-49-0) provides key parameters of NN2000.

The authors, Hønefoss June 25, 2020

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Contents

Chapter 1

Introduction

Over the last two decades, georeferencing of cartographic data has changed from a national/regional to a continental/global perspective. Due to space techniques, especially Global Navigation Satellite Systems (GNSS), a completely new solution for horizontal control networks has been determined.

The Regional Reference Frame Sub-Commission for Europe (EUREF) defined the European Terrestrial Reference System (ETRS) in 1989 (Boucher & [Altamimi,](#page-50-0) [1992\)](#page-50-0). It is based on the definition of the International Terrestrial Reference System (ITRS), and realized through the European Terrestrial Reference Frame (ETRF) at epoch 1989.0. In Norway, this was implemented by GNSS campaigns in 1994-1996. All counties of Norway had changed to the new reference system by spring 2009.

Vertical reference systems realized by precise leveling alone do not allow global solutions. For gravity related heights, often called physical heights, realization of global reference systems may in the future be possible by a combination of GNSS and global geoid models following the definitions of the International Height Reference System (IHRS). However, EUREF has so far focused on leveling and leveling networks to realize physical heights, and has since 1994 worked on the definition and realization of a vertical reference system/frame for Europe, i.e., the European Vertical Reference System (EVRS) realized by the European Vertical Reference Frame (EVRF). See, e.g., Rülke et al. [\(2012\)](#page-52-0) for details on the status of height system unifications in Europe.

The former national height system of Norway, NN1954, was strongly deformed, mainly due to postglacial land uplift [\(Lysaker](#page-51-0) et al., [2006\)](#page-51-0). The lack of correction to an unique reference epoch, caused systematic errors, but with variation depending on the time of realization. The Nordic countries, especially Sweden and Finland faced the same challenges with height systems deformed by land uplift. In order to obtain new and accurate national height systems, cooperation through the Nordic Geodetic Commission (NKG) for a unified height system was initiated in the 1980s when new leveling programs started in all the Nordic countries. The cooperation has resulted in great improvements in the common Nordic leveling network, used in the calculation of EVRF2000, and later in EVRF2007.

The present national height system of Norway is called NN2000. It is, however, not identical to neither EVRF2000 nor EVRF2007. As shown in Chapter [2.2,](#page-8-0) a special Nordic realization was carried out, which NN2000 results from. Thus NN2000 is consistent to the Swedish height system RH2000 and the Finnish N2000.

In Norway, the differences to EVRF2007 varied originally from 0 to 2 cm. Due to new measurements at the west coast of Norway after the release of EVRF2007, the difference to EVRF2007 increased. In Sogn og Fjordane the differences after the final adjustment vary from -4 to 6 cm, and for Møre og Romsdal, Hordaland and Rogaland the differences vary from -3 to 3 cm.

This report describes the fundamental parameters defining NN2000, and how the new heights are realized through the leveling network, the passive geometric network (Landsnettet), and in the map databases.

Chapter 2

Background and evolvement leading to NN2000

Early attempts to establish a height system for Norway were rooted in the 1864 general assembly of the Mittel-Europäische Gradmessung in Berlin. This organization is considered as a precursor of the International Association of Geodesy (IAG), and following its recommendations a tide gauge was mounted in the harbor of Oslo in 1876 to provide a long-term mean sea level reference. Leveling campaigns began in 1887, but the progress was slow. After recording sea level observations for 14 years, a reference marker on the property of the Geographical Survey of Norway in Oslo was established and connected to the tide gauge by leveling. The reference marker was called Normal Null (NN) and served as the fundamental benchmark for all leveling in Southern Norway up until the adjustment of NN1954.

2.1 Short description of NN1954 and NNN1957

In 1916, modern leveling instruments were acquired and the leveling program intensified. The measurements were made along the main communication lines south of the Arctic Circle (at latitude of $66°33'$ N), and in 1953 most of the southern part of the country was covered. In the meantime, tide gauges had been set up at several locations along the coast. The tide gauge records showed different long-term trends essentially due to differences in land uplift along the coast. In Oslo, the uplift rate was found to be about 3 mm/yr. Oslo was thus considered an unsuitable site for a fundamental benchmark, and a new tide gauge was established at Tregde (near the southern extreme of Norway) where the uplift rate was observed to be close to zero.

The leveling network for the southern part of Norway (between 58◦ and 66.3◦ latitude) was adjusted in 1956 and tied to mean sea level determined by seven tide gauges along the coast, each with 22 to 68 years of observations. The tie was obtained by the height of the fundamental benchmark at Tregde above mean sea level. The vertical reference frame was called NN1954, and the results were reported by Trovaag & [Jelstrup](#page-53-0) [\(1956\)](#page-53-0).

A fundamental problem arose in the realization of the height system. While the leveling program had been running for 40 years, post glacial land uplift had systematically deformed the network. No reliable land uplift model existed in 1956, so the adjustment did not take land uplift into account. This also applies to later extensions of the network and lead to a strongly deformed network with no common reference epoch. In some areas, [Lysaker](#page-51-0) et al. [\(2006\)](#page-51-0) found differences of more than 20 cm between the original NN1954 and NN1954 corrected for land uplift.

An additional problem was the lack of observed gravity values along the leveling lines in the 1956 adjustment, needed for calculating geopotential numbers (see Chapter [3\)](#page-12-0). Instead, spheroidal-orthometric corrections were used in order to achieve what was assumed to be orthometric heights [\(Trovaag](#page-53-0) & Jelstrup, [1956\)](#page-53-0). However, [Lysaker](#page-51-0) et al. [\(2006\)](#page-51-0) demonstrated that the spheroidal-orthometric correction is shown to give values closer to normal heights than orthometric heights. The spheroidal-orthometric correction uses Clairaut's formula for gravity instead of observed gravity. Details on the correction can be found in Trovaag & [Jelstrup](#page-53-0) [\(1956\)](#page-53-0) and [Lysaker](#page-51-0) et al. [\(2006\)](#page-51-0).

Due to practical considerations, the leveling network north of the Arctic Circle was originally a separate entity defining a height system called Nord-Norsk Null 1957 (NNN1957) and referred to the tide gauge in Narvik. In 1974, the two networks were connected by a 200 km leveling line from Fauske to Narvik. The difference between the two systems was measured to 28 mm, which was less than the expected accuracy for such a distance. Nevertheless, the name NNN1957 was used until 1 January 1996, when the Norwegian Mapping Authority (NMA) formally decided to use the term NN1954 for both systems and consider them as one common height system for mainland Norway (Statens [kartverk,](#page-52-1) [2009b\)](#page-52-1).

2.2 The Baltic Leveling Ring (BLR2000)

Work on establishing a common European vertical reference frame started in 1945. Following the resolution of the International Union of Geodesy and Geophysics (IUGG) General Assembly in Rome 1954, the network was referred to as the United European Levelling Network (UELN) or Réseau Européen Unifié de Nivellement (REUN). The work resumed in 1994, and four years later EUREF calculated a network consisting of new data together with improved existing datasets and used Normaal Amsterdam Peil (NAP) as fundamental benchmark. The network was denoted United European Levelling Network 95/98 (UELN95/98). At the EUREF symposium in Tromsø in 2000, a new definition of EVRS was adopted followed by a new realization of the UELN called EVRF2000.

Finland and Sweden planned to finish their third national precise leveling by 2001, which was expected to lead to new national height systems. Resolution number 2 of the 14th General Assembly of the Nordic Geodetic Commission (NKG) in October 2002 outlined a common goal for the new national height systems:

The Nordic Geodetic Commission (NKG) recommends the representatives of the national mapping authorities and geodetic institutions in NKG to be active for the adoption of a unified Nordic height system with minimum differences from national height systems and from the European Vertical Datum.

All three countries experienced strong land uplift and faced the same challenges in the treatment of this phenomenon. Additionally, a connection to the fundamental point of the EVRS realization was required to fulfill the resolution. At first glance, a direct adoption of EVRF2000 in the Nordic countries would be the obvious thing to do. However, there were reasons not to do so. First, the connection of the Nordic countries (Norway, Sweden and Finland) to the rest of Europe was weak. Only a single line measured by trigonometric techniques between Denmark and Sweden connected the whole block. Additionally, the UELN95/98 data were not consistent in the treatment of post glacial land uplift. Some countries experiencing land uplift, did not account for this, while others did. Unfortunately, the leveling data were referenced to different epochs, often by using old and uncertain land uplift models. The Nordic countries wanted updated and consistent land uplift corrections for the whole region, and a solid connection between Denmark and Sweden.

The Nordic Geodetic Commission

The Nordic Geodetic Commission (NKG, founded in 1953) is an association of geodesists from Denmark, Finland, Iceland, Norway and Sweden. Its purpose is to give the members possibilities of fruitful gatherings and mutual exchange of professional views and experiences. The NKG is recognized and supported by a number of Nordic organizations, such as the Director Generals of the Nordic Mapping Authorities. In order to forward its vision, the Commission arranges general meetings every four years, and summer schools also every four years, in one of the Nordic countries as the host. NKG is managed by a Presidium and the actual work is done in working groups and working group projects. (Adopted from [www.nordicgeodeticcommission.com\)](www.nordicgeodeticcommission.com)

When the Øresund Bridge between Denmark and Sweden was opened in 2000, the mapping authorities in Sweden and Denmark leveled the connection between the two countries, considerably improving the connection of the Nordic countries to the rest of Europe. There was still a need for an even better connection.

The NKG Working Group for Height Determination (NKGWGH) initiated the work of closing the loop around the Baltic countries in order to have another connection, the Baltic Leveling Ring (BLR). Unfortunately, it was not possible to close the loop around the Gulf of Finland through Russia with leveling data, but alternative methods were used (Mäkinen et al., [2005\)](#page-51-1). A close cooperation between the NKG, all the Baltic countries, the Netherlands, and the UELN computing center was established following a proposal from the NKGWGH to the Technical Working Group of EUREF (Mäkinen et al., [2003\)](#page-51-2). The Nordic countries compiled and screened their new leveling data, tested and adopted land uplift models, and performed regional adjustments. Estonia, Latvia, Lithuania, Poland, Germany, and the Netherlands made their leveling data, stored in the UELN database and used for the EVRF2000 calculation, available to the NKGWGH. Using these data, together with the latest precise leveling data in Denmark, Sweden, Finland, and all precise leveling in Norway (1916-2003), the working group calculated the BLR. All leveling data in Figure [2.1](#page-10-1) were referred to epoch 2000.0 by applying the NKG2005LU uplift model (Agren & [Svensson,](#page-50-1) [2007\)](#page-50-1) prior to the adjustment. The geopotential number (see Chapter [3\)](#page-12-0) at NAP was kept fixed, and the computations were performed in the mean tide system before the result was transformed into the zero tide system. Finally, the resulting geopotential numbers were transformed into normal heights, resulting in the BLR2000 height system $(Mäkinen et al., 2005).$ $(Mäkinen et al., 2005).$ $(Mäkinen et al., 2005).$

The Swedish national height system RH2000 is a subset of the BLR2000 and the Finnish national height system N2000 is a slightly modified version of BLR2000. The calculation of the Norwegian height system NN2000 follows the same procedure as described for BLR2000. The geopotential numbers of the connection points to the Swedish and Finnish network were kept fixed, so NN2000 can be considered as an extension of RH2000

Figure 2.1: The Baltic Leveling Ring (BLR). The dark circle is the fundamental point in Amsterdam, Normaal Amsterdam Peil (NAP).

and N2000. Further details on the calculation of NN2000 are presented in Chapter [5.](#page-25-0)

2.3 Further developments in Europe

After EVRF2000 was published, more countries have added data to the UELN and several countries have provided new data, e.g., the Netherlands and the Nordic countries. During the work with BLR2000, the problems concerning postglacial land uplift and a common reference epoch were thoroughly addressed and revealed shortcomings in the EVRF2000. Enhanced EVRS conventions and parameters were needed, and a new realization EVRF2007 was released. The main differences between EVRF2000 and EVRF2007 are summarized below. For more details see [Sacher](#page-52-2) et al. [\(2009\)](#page-52-2).

- The datum point of EVRF2000 (000A2530) was not included in the new national leveling network of the Netherlands and is therefore no longer available as a datum point. In order to keep the level of the EVRF2000 datum, EVRF2007 is realized by 13 datum points in which the difference to EVRF2000 in sum is set to zero.
- In EVRF2000, the data from Finland, Norway and Sweden were reduced to the

epoch 1960, while the other data had not been corrected to a common epoch. For EVRF2007, the common reference epoch is 2000.0. All data within the coverage of the NKG2005LU model (Agren & [Svensson,](#page-50-1) [2007\)](#page-50-1) have been corrected with this model.

• The EVRS definition of a zero tide system was not realized in EVRF2000, the tide system was mixed and unknown. The tide system of the national leveling data has been clarified and EVRF2007 is uniformly reduced to the zero tide system.

EUREF adopted EVRF2007 as the new realization of the EVRS at the EUREF symposium in Brussels, June 2008, after the adjustment of NN2000 was finished. The definitions and realizations of EVRF2000 thus form the basis of NN2000, as well for the Swedish RH2000 and the Finnish N2000. The establishment of EVRF2007 aimed at keeping the differences to EVRF2000 small. The differences between EVRF2007 and NN2000 were between 0 and 20 mm throughout Norway, the NN2000 heights always higher than EVRF2007 heights. After the recalculation of the western part, however, the differences are higher due to new important measurements in the county of Sogn og Fjordane after the release of EVRF2007 (see Chapter [5\)](#page-25-0).

Chapter 3

Theoretical baseline for NN2000

In order to properly define a vertical reference system, four choices have to be made:

- 1. Type of heights
- 2. Reference epoch and land uplift model^{[1](#page-12-2)}
- 3. Zero level
- 4. Permanent tide system

For NN2000, the choices must agree with the definition of EVRS2000 as determined by the EUREF Technical Working Group [\(Augath](#page-50-2) & Ihde, [2002\)](#page-50-2).

Definition of the national vertical reference system NN2000: NN2000 is a zero tide vertical reference system tied to NAP at epoch 2000.0. The NKG land uplift model (NKG2005LU) is applied. The vertical reference system is realized through normal heights at 19000 first order benchmarks throughout the country.

The four choices are addressed below.

3.1 Height type

Precise height determination over large areas must be based on geopotential numbers, since leveling alone does not yield unambiguous height values. This is owing to the nonparallel equipotential surfaces of the Earth's gravity field [\(Hofmann-Wellenhof](#page-51-3) & Moritz, [2005\)](#page-51-3). The geopotential number (C) at point A is defined as the difference between the gravity potential at the geoid (W_0) and at the point A (W_A) .

$$
C = W_0 - W_A = \iint_0^A g \, dn \tag{3.1}
$$

Here q is the observed gravity and dn the leveled height difference [\(Hofmann-Wellenhof](#page-51-3) $\&$ [Moritz,](#page-51-3) [2005,](#page-51-3) p. 159). An accuracy of 10^{-6} m/s² (0.1 mGal) on g is sufficient for surface gravity observations along the leveling lines [\(Torge,](#page-52-3) [1989,](#page-52-3) p. 91).

¹Mainly for regions experiencing land uplift

From the geopotential numbers, heights of different types are derived [\(Hofmann-](#page-51-3)[Wellenhof](#page-51-3) & Moritz, [2005,](#page-51-3) p. 168).

$$
height = \frac{C}{G} \tag{3.2}
$$

The type of height obtained depends on the choice of gravity (G) . If mean gravity along the plumb line is used, orthometric heights are achieved, while the use of mean normal gravity yields normal heights. NN2000 gives normal heights. The definition and description of orthometric heights are included to better see the difference.

3.1.1 Orthometric heights

The orthometric height is defined as the distance from the geoid along the curved plumb line to the point of interest. From Equation [\(3.2\)](#page-13-2), orthometric heights are given [\(Hofmann-](#page-51-3)[Wellenhof](#page-51-3) & Moritz, [2005,](#page-51-3) Equation 4-27):

orthometric height =
$$
H = \frac{C}{\overline{g}}
$$
, (3.3)

where \bar{q} is mean gravity along the plumb line. On the geoid, the orthometric height equals zero. In order to calculate orthometric heights from geopotentials, the mean gravity along the plumb line has to be known. Real mean gravity values are impossible to obtain since the density distribution of the Earth is only approximately known, and it is difficult to measure gravity inside the Earth. Thus, orthometric heights are always approximated. Almost exclusively, Helmert heights [\(Hofmann-Wellenhof](#page-51-3) & Moritz, [2005,](#page-51-3) p. 163) are used as an approximation to strictly defined orthometric heights. [Helmert](#page-51-4) [\(1890\)](#page-51-4) used the Poincar´e and Prey gravity gradient to evaluate the mean gravity value halfway down the plumb line from observed gravity at the Earth's surface. Poincaré and Prey reduction assumes normal gravity and a Bouguer plate of constant density. Hence, Helmert heights are based on three assumptions: 1) gravity is behaving linearly between the geoid and the surface; 2) constant density; and 3) fixed free-air gradient. [Tenzer](#page-52-4) et al. [\(2005\)](#page-52-4) have defined a more rigorous orthometric height, in that the mean gravity along the plumb line is evaluated more accurately.

3.1.2 Normal heights

In order to avoid dealing with the unknown mean gravity along the plumb line, Molodensky formulated the theory of normal heights in 1945. That is, "orthometric heights" in a normal gravity field. This means that actual mean gravity is replaced by normal mean gravity $(\overline{\gamma})$, i.e., the mean of normal gravity between a reference ellipsoid and the telluroid [\(Hofmann-Wellenhof](#page-51-3) & Moritz, [2005,](#page-51-3) Equation 4-61):

normal height =
$$
H^N = \frac{C}{\overline{\gamma}}
$$
 (3.4)

The reference surface is then a mathematical ellipsoid instead of the physical geoid. The advantage with normal gravity is that the formula is easily evaluated without approximations. The physical meaning however, is not that obvious. If the Earth's gravity potential at a point P is W_P , then there is a point Q on the plumb line where the normal potential

Figure 3.1: Principal sketch of normal heights.

 U_Q equals the actual potential $W_P = U_Q$. The normal height is then the distance from the ellipsoid to point Q , see Figure [3.1.](#page-14-1) All points Q define the telluroid. The telluroid is an approximation to the Earth's surface, the topography of a "normal Earth", but it does not mirror the actual topography. If the normal height is deposed from point P along the plumb line, the normal heights define another surface, the quasi-geoid. The quasi-geoid may also be regarded as a reference surface for normal heights. As a rule of thumb, the difference between the geoid and the quasi-geoid, or equally orthometric and normal heights, is 0.1 times the square of the height in kilometer [\(Hofmann-Wellenhof](#page-51-3) $\&$ [Moritz,](#page-51-3) [2005\)](#page-51-3).

3.2 Reference epoch and postglacial land uplift

Many countries hardly experience any vertical land motion. Norway and the other Nordic countries however, are located in the Fennoscandia uplift area. During the last ice age, the Earth's crust was deformed due to the weight of the ice masses. When the ice melted, the elastic crust started to rebound to its pre-deformed position. This rebound is slow because of the viscosity of the Earth's mantle. Fennoscandia is an area exposed to postglacial rebound and several models describing the vertical motion are available. [Ekman](#page-50-3) [\(1991\)](#page-50-3) gives a review of some of the scientific work on the subject. There are different approaches to calculating the present-day uplift field. Geophysicists use the theory on how the Earth responses to changes in ocean and ice loads to obtain their land uplift models, while geodesists obtain empirical models from observations from tide gauges, leveling, and lately permanent GNSS stations. The NKG land uplift model (NKG2005LU) shown in Figure [3.2](#page-15-1) is a combined model. A smoothed version of the empirical model of [Vestøl](#page-53-1) [\(2006\)](#page-53-1) is merged with the GIA model of [Lambeck](#page-51-5) et al. [\(1998\)](#page-51-5). Further details on the smoothing and combination may be found in $\text{Agren} \&$ [Svensson](#page-50-1) [\(2007\)](#page-50-1).

Due to land uplift, leveling data have to be corrected to a common epoch to obtain a

Figure 3.2: The land uplift model NKG2005LU. The isolines indicate the estimated vertical velocity in millimeters per year relative to mean sea level (1892-1991). Outside the -2 mm/yr isobar, the value is set to the constant -2 mm/yr.

consistent vertical reference frame. To minimize the influence of errors in the uplift model, it would be advantageous to choose the mean epoch of the leveled data or an epoch close to it. On the other hand, from practical considerations an epoch as close to current time as possible is desired. As a compromise, the epoch 2000.0 was selected.

3.3 Zero level

Since 1860, most countries in Europe have realized vertical reference systems based on national precise spirit leveling networks. They are in most cases related to mean sea level at one or more tide gauges and realized through some kind of gravity-related heights [\(Augath](#page-50-2) & Ihde, [2002\)](#page-50-2). Orthometric heights refer to the geoid. Normal heights refer to the reference ellipsoid. Thus, today the zero level of a vertical reference system is realized through a reference marker with known height or geopotential number. That is, the gravity potential W_0 is set equal to the normal geopotential U_0 for a mean Earth. In order to follow the resolution of the NKG General Assembly from October 2002, the realization of the zero level for NN2000 is equal to the zero level of the EVRS, which is also the zero level of the Swedish vertical reference system, RH2000 (\AA gren & [Svensson,](#page-50-1) [2007\)](#page-50-1) and the Finnish vertical reference system N2000. The realization follows the regulations of [Augath](#page-50-2) & Ihde [\(2002\)](#page-50-2):

The vertical datum of the EVRS is realized by the zero level through the Normaal Amsterdam Peil (NAP). Following this, the geopotential number in the NAP is zero:

$$
C_{NAP}=0
$$

• For related parameters and constants the Geodetic Reference System 1980 GRS80 is used. Following this the Earth's gravity potential through NAP (W_{NAP}) is set to be the normal potential of the GRS80.

$$
W_{NAP} = U_{GRS80}
$$

• The EVRF2000 datum is fixed by the geopotential number and the equivalent normal height of the reference point 000A2530/13600 of the UELN.

The zero level of NN2000 is in other words the zero level of benchmark 13600 in the UELN numbering system. This zero level is 0.71599 m below the top of the benchmark and is the exact NAP reference.

3.4 Permanent tide system

The Earth is affected by the gravitational attraction from celestial bodies, mainly the Moon and the Sun. The attraction is dependent on the position of the celestial bodies and thus periodic. The effect on the Earth's crust is called Earth tides. Gravitational attraction may be expressed in terms of a potential, and for the celestial bodies it is called the tide generating potential. It deforms the Earth's crust, and has a perturbing effect directly on the Earth's gravity potential. Tidal effects influence local gravity and are detected in gravity observations. The effect may be split in two terms, one is due to the direct change in the gravity field. Secondly, the observed gravity will change because the height has changed due to the deformation of the crust.

The long time mean of the tidal effects is called the permanent tide. Thus, the tide generating potential may be divided into a permanent and a periodic part [\(Ekman,](#page-50-4) [1989\)](#page-50-4). Gravity data are utilized for both height realization and geoid determination. To avoid confusion, it is important to handle the permanent tidal effects consistently. There are three different geoid definitions; mean tide, zero tide, and tide free [\(Torge,](#page-52-5) [2001\)](#page-52-5).

1. Mean tide geoid: The gravitational effect of the permanent tidal potential is kept

in the gravity observations. Corresponds to how the geoid and the crust actually behave in the long time mean.

- 2. Zero tide geoid: The gravitational effect of the permanent tidal potential is split in two terms, one direct that is due to the lunisolar attraction (Moon and Sun) and one indirect due to the deformation of the Earth. The direct effect is eliminated and the indirect effect is kept in the gravity observations. Corresponds to the crust in the long time mean, and the geoid if we assume there is no Moon or Sun, but still with a deformed crust.
- 3. Tide free geoid: The gravitational effect of the permanent tidal potential is eliminated from the gravity observations. Corresponds to a geoid and a crust assuming there is no Moon or Sun.

According to [Augath](#page-50-2) & Ihde [\(2002\)](#page-50-2), EVRS has adopted the zero tide geoid, as has the BLR 2000. Thus, the zero tide system was chosen for NN2000 as well.

Chapter 4

Data and measuring methods

Within Europe, precise leveling data still are the preferred data source for realizing national and regional height systems (Rülke et al., 2012), and NN2000 is no exception. Additionally, due to the rough terrain in Norway, fjord crossings are needed to build up a network of closed loops. Since leveling alone does not yield unambiguous height values, reliable gravity data are important to obtain geopotential differences. This chapter describes the data needed and used for realizing NN2000.

4.1 Precise leveling

When NN2000 was first realized in 2008, the leveling network consisted of 26.000 km of precise spirit leveling data form 1916 to 2008. For the final adjustment in 2012, some more kilometers were added, while a few old lines were rejected. This is further described in Chapter 6. Within the NKGWGH, the Nordic countries first agreed upon common guidelines for precise leveling in 1984. The leveling took many years to complete and the original guidelines were slightly modified in all countries. [Erikson](#page-50-5) et al. [\(2014\)](#page-50-5) describe in detail the original guidelines and the different modifications in all countries.

Precise leveling is performed by double leveling, i.e., all lines are leveled back and forth. The measuring procedure involves one instrument and two invar leveling staffs. The maximum allowed distance between the instrument and the staffs are 50 m and the leveling is performed in sections, where a section has a marked benchmark in both ends. The difference between the distance of the foresights and the backsights for one section should not exceed 5 m. Temperature is measured at the start and at the end of a section, in order to correct for the invar string's thermal expansion. A brief historical overview of the NMA's precise leveling data is summarized in Table [4.1,](#page-19-0) where some milestones are outlined for each period. Figure [4.1](#page-20-2) provides an overview of kilometers of double-run leveling measured per year from 1952 to 2016.

Following international recommendations, the maximum accepted difference between the foresight and backsight measurements was in 1972 reduced from 4 to 2 mm multiplied by the square root of the distance in kilometers. The observations before and after 1972 are referred to as the old and new data, respectively.

Table 4.1: Overview of precise leveling carried out by the Norwegian Mapping Authority. Requirement is the highest accepted difference between the foresight and backsight measurements and s is the distance in kilometers.

Period	Instruments	Req.	Remarks
$1916 - 1953$	Levels with optical micrometer. 1919 - 1946: Zeiss levels. From 1946: Wild N-3 levels. Staff's scale on invarstrings. Normal meter of invar for calibrating the staffs. Foot leveling.	$4\sqrt{s}$ mm	With one exception, all lines are measured in the southern part of Norway. All existing lines before 1916 were releveled. On average 250 km were leveled, in both directions, each year. The normal meter was calibrated to the international standard meter.
$1954 - 1979$	Instruments with compensator pendulum in the end of the period. Foot leveling.	$4\sqrt{s}$ mm From 1972: $2\sqrt{s}$ mm	The leveling network was extended to the northern part of Norway. One line, from Fauske to Narvik, connected the northern network with the southern in 1974.
1980 - 1996	Motorized leveling. In average, the production increases from 5 to 10 km single run leveling per day. The staffs were calibrated at the calibration basis at Lantmäteriet in Sweden every year.	$2\sqrt{s}$ mm	Start of cooperation in the 1980s with Lantmäteriet on leveling in the area close to the border. Plans for extending the leveling network to as many municipalities as possible.
1997 \rightarrow	Digital levels. Foot leveling only. The staffs were calibrated at in-house calibration basis every year.	$2\sqrt{s}$ mm	The main motivation for leveling was to establish a dense and even distribution of GNSS/leveling points.

Figure 4.1: Kilometers of double-run leveling measured per year from 1952 to 2016.

4.1.1 The leveling network

The Norwegian precise leveling data consist of single lines from 1916 to present, and almost all data are needed to form a network covering the entire country. As shown in the left part of Figure [4.2,](#page-21-0) there are single lines that are releveled once and twice, but without forming a network.

The Norwegian data can not be divided into a first, second or third order leveling network like in other Nordic countries. Still, the precise leveling data stored in the database of NMA are divided into first order and second order data. The second order data amount to 1100 km. Today, the classification criteria is unclear, since the instruments, rejection limits, and other requirements appear to be the same as for the first order leveling. Nevertheless, the classification is preserved for historical reasons.

Additionally, leveling data from the Norwegian National Rail Administration (called Norges Statsbaner, NSB, at the time of observation) are stored in NMA's database. This leveling was accomplished during the 1960's, the 1970's and the 1980's, and heights were measured along the railways on benchmarks established every 500 m. The railway lines have been connected to first order benchmarks close to the tracks. The railway leveling data amount to 3680 km and 7180 benchmarks. The second order leveling data and the railway data are shown in the right part of Figure [4.2.](#page-21-0)

The first leveling lines have benchmarks every third kilometer, newer lines have an approximate spacing between the benchmarks of 1 km. By 2008 the first order leveling network consisted of 19000 benchmarks.

4.2 Fiord crossings

Leveling lines should form closed polygons or loops for control. Due to long fjords and high mountains, this is often difficult in Norway. To obtain control, it has been necessary to cross fjords where there is no tunnel or bridge. This requires the use of other measuring

Figure 4.2: Left: Norwegian first order leveling network. Red lines are measured once, blue lines are measured twice, and green lines are measured three times. Right: The railway leveling network (red) and the second order leveling lines (green) throughout Norway.

techniques than ordinary leveling. During the years, different techniques have been in use. Unfortunately all of these techniques suffer from lower accuracies than leveling because of refraction and geoid variations. Up to 1995, we used leveling instruments and special targets on the staffs. Later, accurate total stations have been used.

The problems with refraction were reduced by simultaneous measurements from both sides of the fjord. Additionally, bad weather conditions were avoided. The best weather for fjord crossing measurements is when there is as little sun as possible, preferably clouded with no rain or strong wind.

Geoid variations are complex and may give an unreliable result. It is necessary to assume that the deflection of the vertical is either the same on both sides of the fjord or the same value, but with opposite sign. If not, the result will be systematically wrong. No measurement of the deflection of the vertical has been performed, so we do not know if this assumption is fulfilled. However, if the geoide changes irregularly over the fjord, these requirements alone may not be enough to avoid systematic errors.

In addition to simultaneous measurements, it has been normal procedure to swap the instruments, including the observer, one or several times during the observation campaign. The height difference is then calculated for each setup. It turns out that the result often changes systematically depending on which side the instrument is located. This indicates

Figure 4.3: All fjord crossings in the first order leveling network. The pink dots show crossings later replaced by leveling through a tunnel or across a bridge. Figure [4.4](#page-23-0) shows a more detailed view of the area within the blue rectangle.

that instrument and human errors influence the measurements.

Altogether, there are 116 fjord crossings in the first order leveling network. They are shown in Figure [4.3.](#page-22-0) Three of them, (marked in pink in Figure [4.3\)](#page-22-0) were replaced with ordinary precise leveling when new bridges or tunnels opened. Usually there is only one fjord crossing in the same loop, but a few loops have two crossings. Unfortunately, there is one case with ten fjord crossings in the same loop as shown in Figure [4.4.](#page-23-0)

The quality of the leveling network is degraded due to all the fjord crossings. In particular, long crossings are unfortunate. As seen in Table [4.2,](#page-23-1) the average distance of the ten longest crossings are 4.4 km and in total 24 are longer than 3 km. For future height systems it is important to quality-assess the fjord crossings. Combination of GNSS and a geoid model may contribute to this, as well as local tide gauge measurements.

Table 4.2: Statistics of the fjord crossings.

Total number	116
Average distance (m)	2004
Median distance (m)	1741
Average distance of the 10 longest (m)	4401
Number of crossings longer than 3 km	24

Figure 4.4: Map of leveling loop in Sunnfjord with ten fjord crossings.

4.3 Gravity data

In addition to the leveled height difference, gravity is needed to determine the geopotential difference between two benchmarks. As long as the distance between the benchmarks is within a few kilometers, it is sufficient to know the gravity at the benchmarks and use the mean value for the entire leveled section [\(Hwang](#page-51-6) & Hsiao, [2003\)](#page-51-6). From around 1950 to 2000, gravity has been measured directly on most of the benchmarks along the leveling lines and usually the same year as they were leveled. The gravity measurements have been relative measurements observed in closed loops, starting and ending at a gravity benchmark point (basis point). Several LaCoste & Romberg (LCR) instruments have been used.

From the end of the 1960's, a new gravity network for geoid determination was established and the leveling lines, although still measured, had second priority. The gravity network consists of a basis network with approximately 280 points marked with a benchmark and a relative network with measurements every 5 km covering all of Norway, approximately 7000 points. In addition to the NMA measurements, the Geological Survey of Norway (Norges Geologiske Undersøkelse, NGU) has collected a large amount of gravity data (approximately 65000 points). NGU has connected all their gravity data to the basis network. Since 1990, NMA and NGU store all gravity values in a common Norwegian gravity database.

Although gravity has not been measured at the benchmarks of new leveling lines after 2000, the gravity network is still densified by relative measurements with LCR instruments, and more recently by Scintrex CG5 instruments. In addition, absolute gravity has been observed at around 20 sites with instruments of the FG5 type [\(Breili](#page-50-6) et al., [2010;](#page-50-6) [Ophaug](#page-52-6) et al., [2016\)](#page-52-6) and at approximately 250 sites with A10 instruments. The gravity database makes it possible to interpolate gravity with sufficient accuracy in the benchmarks not measured. For the calculation of NN2000 we have used interpolated values for all benchmarks.

Chapter 5

Calculation of NN2000

The calculation and implementation of NN2000 heights consist of two parts, the leveling network and the passive GNSS network. This chapter describes the calculation of the leveling network, and Chapter [6](#page-38-0) describes the determination of heights in the passive GNSS network (Landsnettet).

The calculation of the leveling network was carried out in several steps, which are described in the sections below. First, the observations were screened for outliers as part of the land uplift determination [\(Vestøl,](#page-53-1) [2006\)](#page-53-1). Second, the leveling network was adjusted using geopotential differences and least squares adjustment (LSA). The LSA was done in three steps. In the first step, the geopotential numbers from the BLR adjustment described in Section [2.2](#page-8-0) at common points along the Swedish and Finnish border were held fixed. In the last two steps, the result from the previous step(s) were kept fixed. All three steps were carried out using the commercial adjustment software Gemini Oppmåling (version 5.4).

The adjustments were finished in 2008, but the western part of Norway showed large misfits. The results for the western part were thus considered as preliminary. After collecting more leveling data and controlling several fjord crossings, the final NN2000 adjustment was done in 2012, with only height values in the four westernmost counties changing.

5.1 Preparations for the adjustment

5.1.1 Geopotential numbers

Geopotential differences were used in the adjustment of the leveling network. The geopotential difference (C_{AB}) between two points A and B was obtained from leveled height differences (dn_{AB}) and interpolated gravity values $(g_A \text{ and } g_B)$ according to

$$
C_{AB} = dn_{AB}\frac{g_A + g_B}{2} \tag{5.1}
$$

In Equation [\(5.1\)](#page-25-3), gravity should be in kilogal (10 m/s²). This means that the height differences generally are multiplied with a number varying around 0.98. The result is a geopotential number with unit g.p.u. (geopotential unit).

5.1.2 Permanent tide and reference epoch

Both leveled height differences and gravity have to be in the same tide system. Since we do not apply any tidal correction to our leveling observations, they are referring to the mean tide system. Periodic tidal corrections were applied to the gravity data, but no permanent tide corrections. This results in mean tide values for gravity as well. Consequently, the LSA of the leveling network was done in the mean tide system, and the adjusted values were converted to the zero tide system afterwards.

Additionally, all leveling observations need to be corrected to a common reference epoch. We used the land uplift model NKG2005LU (\AA gren & [Svensson,](#page-50-1) [2007\)](#page-50-1) to correct the observations from the observation epoch to 2000.0. Since some leveling lines go back to 1916, this correction is essential in order to fulfill the requirement of a common reference epoch for the whole network.

5.1.3 Outlier detection

The leveling lines are shown in Figure [5.1.](#page-28-1) The data set includes the railway leveling lines as well, since they all are connected to NMA's first order leveling network and are used for land uplift determination. Before the adjustment, outliers were detected and removed from the data. We used multiple Student's t-test, which implies that an outlier (∇) is estimated for each observation, one at a time. Then test values can be calculated by dividing the outlier on its standard error (s_{∇}) [\(Pelzer,](#page-52-7) [1985;](#page-52-7) [Revhaug,](#page-52-8) [1989\)](#page-52-8):

$$
t = \frac{\nabla}{s_{\nabla}},\tag{5.2}
$$

Following [Revhaug](#page-52-8) [\(1989\)](#page-52-8), a t-value higher than three was set as rejection criterion. Sometimes suspicious lines consist of smaller parts that individually cannot be rejected based on their t-value. However, by removing the entire line and performing a Fisher test based on the reduction of the weighted sum of squared residuals, we may sometimes reject the entire line as an outlier. This is a possibility, even when the smaller parts individually do not exceed the rejection limit (see [Revhaug](#page-52-9) [\(2007\)](#page-52-9) for further details).

Outliers cannot be detected without considering the land uplift. Since the leveling observations are important input to the land uplift calculation, the outlier test was performed as part of the land uplift determination, as described in [Vestøl](#page-53-1) [\(2006\)](#page-53-1).

The outlier test identified 13 first order leveling lines, partly or completely, as listed in Table [5.1.](#page-27-0) Additionally, four lines in the railway network listed in Table [5.3](#page-31-2) were rejected. The rejected lines are not used in the first step (Section [5.2\)](#page-28-0), the adjustment of the nodal points. In step two (Section [5.3\)](#page-31-0), and step three (Section [5.4\)](#page-31-1), the rejected first order lines and railway lines, respectively, are again included.

5.1.4 Weighting strategy

The observations were assigned weights proportional to the inverse of the leveled distance using Equation [\(5.3\)](#page-26-3)

$$
w_i = \frac{1}{s_0^2 d_i},\tag{5.3}
$$

where w_i is the weight of observation i, s_0 is the standard error for 1 km leveling, and d_i is the leveled distance in kilometers. s_0 was set to 1.34 mm for observations prior to

Line number	Obs. year	Outlier (cm)	t -value	Description	Remark
307	1953	10.2	4.7	Alta - Kautokeino	
327	1982	8.3	4.5	Sortland - Fiskebøl	Suspicious fjord crossing over Hadselfjorden
228	1984	-8.8	4.3	Brønnøysund - Leirfjord	
85	1935	7.5	4.1	Nybergsund - Sørvollseter	
250	1987	8.1	3.8	Rutledal - Leirvik $(part of line 250-1987)$	Suspicious fjord crossing over Sognefjorden
39	1942	5.8	3.7	Gol - Borlaug	
250	1987	-6.8	3.6	Gjølanger - Vevring $(part of line 250-1987)$	Crossing Dalsfjorden and Førdefjorden
70	1944	5.1	3.3	Tonstad - Sinnes $(part of line 70-1944)$	From point C38N0019 to point C38N0043
289	1989	-2.8	3.2	Kjenn - Drøbak $(part of line 289-1989)$	From point G36N0216 to point G35N0113
223	1976	3.0	3.3	Lærdal - Revsnes - Kaupanger	Suspicious fjord crossing over Sognefjorden
101	1990	3.2	$4.7*$	Fannrem - Heimsjø	
11	1998	2.6	3.1	Nesodden - Bekkelaget	Student work including a 5 km long fjord crossing
265	1990	5.4	$3.8*$	Støren - Rørås	
203	1957	4.5	3.1	Mo - Umbukta	

Table 5.1: Rejected first order leveling lines. We rejected all lines with a t-value larger than three. Stars (*) in column four indicate that we used the Fisher-test instead of the Student's t-test.

1972, and to 1.12 mm after 1972, according to variance component estimation (see [Vestøl](#page-53-1) (2006)).

Additionally, observations were assigned lower weights if they included one or more fjord crossings. Assuming vertical angles over fjords with an accuracy of $\alpha = 0.2$ mgrad, the corresponding accuracy of the height difference (s_f) is

$$
s_f = d_f \cdot \sin \alpha,\tag{5.4}
$$

where d_f is the distance over the fjord. The total accuracy (s_T) for the line is then

$$
s_T = \sqrt{\frac{A_0^2 d_i + s_f^2}{6}},\tag{5.5}
$$

and the corresponding weight:

$$
\frac{1}{\sqrt{\frac{\hat{d}_0 d_i + s_f^2}{n}}} \tag{5.6}
$$

5.2 Step one: Adjustment of nodal points

The leveling data were organized into lines between nodal points, i.e., points where the lines in the leveling network intersect. For each line, the geopotential differences between benchmarks were summed up to a geopotential difference between the nodal points. After removal of outliers, a LSA of the geopotential differences was performed, keeping the geopotential numbers along the border fixed (the red points in Figure [5.1\)](#page-28-1).

Figure 5.1: The first order leveling network and the railway leveling network organized as lines between nodal points. Green dots: Nodal points with unknown geopotential number. Red dots: Points with known geopotential number from the BLR adjustment.

5.2.1 Discussion on weights

Using the weighting strategy described in Section [5.1.4,](#page-26-2) the a posteriori standard deviation of the unit weight from the adjustment in Gemini Oppmåling is 1.11 . It is arguable that the old observations should have been assigned smaller weights since they are affected by the uncertainty of the uplift model. Another issue regarding weighing is the fact that we have calculated the total accuracy of two or more fjord crossings in the same line, just by simply adding the standard errors. From the law of error propagation, the correct procedure for calculating the accuracy is to calculate the square root of the sum of squared errors. This blunder was done unintentionally and it was not discovered before writing this report. However, the effect is small compared to the accuracy of the result.

5.2.2 Quality of the leveling network

To get an impression of the quality of the leveling network, the residuals from the adjustment listed in Appendix [B.2](#page-62-0) and the loop misclosures may be examined. The loop misclosure is the sum of all geopotential differences in a closed polygon or leveling loop. If there is no error and we correct for land uplift, this sum ought to be zero. However, the misclosure for many loops is far from zero. Figure [5.2](#page-29-2) shows the loops with misclosure exceeding the assumed measuring error with a factor of three, i.e., the loop misclosure is three times higher than 1 mm multiplied by the square root of the leveled distance in kilometers. 9 of 114 loops exceed this limit. Table [5.2](#page-30-0) lists the 19 loops with the highest loop misclosure, including those shown in Figure [5.2.](#page-29-2)

Figure 5.2: Leveling loops that exceed or are close to the limit for maximum accepted misclosure. The limit is set to 3 mm multiplied by the square root of the leveled distance in kilometers (3 mm $\sqrt{d(km)}$). The numbers identify the loops listed in Table [5.2.](#page-30-0)

Sometimes one or more lines in a loop are releveled once or twice. In such cases, the average value is used when calculating the misclosure. It might be difficult to determine the specific line causing a high misclosure. Since the outlier detection previously described also solved for an unknown land uplift, it is challenging to separate land uplift from measurement errors, especially when the redundancy is low. When calculating the misclosure, the land uplift model NKG2005LU (\AA gren & [Svensson,](#page-50-1) [2007\)](#page-50-1) has been used to correct the observations to the reference epoch 2000.0.

The standard errors of the adjusted geopotentials listed in Appendix [B](#page-60-0) range from 0.001 to 0.023 g.p.u. In general, the standard error increases with the distance from the known points located along the Swedish and Finnish border. These standard errors do not say anything about the relative accuracy between points. Additionally, they are to some extent misleading since we did not reduce the weight of the oldest measurements, which are influenced by the uncertainty of the land uplift model. However, they give a general indication of the quality of the network, showing that big residuals degrade the accuracy.

5.3 Step two: Adjustment of the first order network

The first order network was determined using LSA, where the geopotential numbers of the nodal points were kept fixed. The rejected lines from the outlier detection in the first step were again included, now as geopotential differences between neighboring benchmarks in the line. As a consequence, the sum of squared residuals will scale up and increase the standard error of the calculated geopotential numbers in this step. However, this will not affect the determined geopotential number in any other point than those in the rejected line.

When a benchmark is situated on anything but bedrock, old observation(s) connected to the point are rejected if instability has been proven. This was done automatically when the observations were exported from the leveling database into Gemini.

The weighting strategy in the second step was the same as in the first. The reduced weights of the fjord crossings have now the effect that these observations get a bigger part of the residuals and reduce the errors on land correspondingly.

The adjustment included 18823 unknown points. In addition, we had 414 nodes with known geopotential number from the first step. The number of observations were 21205, which gave 2382 degrees of freedom. The standard errors from this adjustment is of little interest since they do not reflect the uncertainty of the nodal points from the first step, and as mentioned, they are scaled up because lines rejected in the first step now are included.

5.4 Step three: Adjustment of the second order and the railway networks

The railway network was first organized as lines between nodal points in the first order network and became part of the outlier detection. Table [5.3](#page-31-2) lists the rejected lines, and indicate outliers up to 8 cm.

Table 5.3: Rejected railway-leveling lines. We rejected all lines with a t value higher than three.

The rejection of these lines in the first step had no implications on the adjustment in the third step. However, in the outlier analysis we rejected these lines, and this information is important to get a correct picture of the accuracy of the railway leveling. Nevertheless, we used the lines in the third step keeping the first order points fixed.

The second order network is not really a network. As shown in Figure [4.2,](#page-21-0) it consists of short lines only. Together with the railway network, these lines were calculated in a common LSA, using geopotential numbers and the same weighting strategy as for the first step.

5.5 From geopotential numbers to height values

The adjustments in all three steps were based on geopotential numbers and differences obtained by Equation [\(5.1\)](#page-25-3). The Norwegian height system NN2000 should, according to its definitions, give normal heights in the zero tide system. Therefore, the resulting geopotential numbers (with unit g.p.u) from all adjustments were converted to normal heights in meters, and transformed from the mean tide system to the zero tide system.

The transformation between the mean and zero tide system followed Equation [\(5.7\)](#page-32-1) [\(Ekman,](#page-50-4) [1989\)](#page-50-4)

$$
C_{\rm ZT} = C_{\rm MT} - 0.296 \cdot (\sin^2 \varphi_N - \sin^2 \varphi_S),\tag{5.7}
$$

where C_{ZT} is the geopotential number in the zero tide system, C_{MT} is the geopotential number in the mean tide system, φ_S is the latitude of Normaal Amsterdam Peil (NAP), and φ_N is the latitude of the point of interest. Note that Equation [\(5.7\)](#page-32-1) is not strictly correct for geopotentials. The factor 0.296 should have been multiplied by $\bar{\gamma}$. Unfortunately, this blunder was discovered during the preparation of this report and leads to a systematic error of about 0.7 mm in southern and 1.5 mm in northern Norway. We think such a small error will not be significant for the users.

Normal heights were then obtained by Equation [\(3.4\)](#page-13-3) with $C = C_{ZT}$:

$$
H^N = \frac{C_{\rm ZT}}{\bar{\gamma}}.\tag{5.8}
$$

Following [Ihde](#page-51-7) et al. [\(2002\)](#page-51-7), the mean normal gravity along the normal plumb line is given

$$
\bar{\gamma} = \gamma - \frac{0.3086H + 0.000000072H^2}{2},\tag{5.9}
$$

where γ is the normal gravity at the reference ellipsoid. Since we do not know the height H exactly, we iterate Equation [\(5.8\)](#page-32-2) and [\(5.9\)](#page-32-3) three times and substitute H with H^N for each iteration (see [Hofmann-Wellenhof](#page-51-3) & Moritz [\(2005,](#page-51-3) Section 4.4)).

The normal gravity at the ellipsoid is conventionally determined by [\(Moritz,](#page-51-8) [2000\)](#page-51-8)

$$
\gamma = 978032.67715 \quad \frac{1 + 0.001931851353 \sin^2 \varphi}{\sqrt{1 - 0.0066943800229 \sin^2 \varphi}} \Bigg) \Bigg(\tag{5.10}
$$

which is based on Somigliana's closed for mula.

The results from all steps in the adjustments are stored in the leveling database of NMA, both the final geopotential numbers and the normal heights in both the mean and the zero tide systems. Additionally, geopotential numbers and normal heights in the tide free system are determined and stored for the sake of completeness. Thus, every point has three different normal heights and three different geopotential numbers.

5.6 The 2012 adjustment

The adjustments described so far were finished in 2008, but the western part of the country showed large misfits. The loop crossing Sognefjorden, as shown in Figure [5.3,](#page-33-1) had a misclosure of 10.5 cm in the 2008 adjustments, which was the largest in the network. We suspected errors in the westernmost fjord crossing, and opened the loop by rejecting that measurement in the adjustment (this is the reason why this loop is not marked in Figure [5.2\)](#page-29-2). We were not satisfied with this solution, since we could not prove that this was the only possible explanation of the large misclosure. Moreover, eliminating the fjord crossings strongly influenced the result for the entire western part of the network. It was decided that the 2008-adjustment should still be retained, but for the four western counties (Rogaland, Hordaland, Sogn og Fjordane, and Møre og Romsdal), the result was regarded as preliminary.

A new adjustment was planned to take place when we had carried out more measurements. From 2009 to 2011, we repeated the western and eastern fjord crossing over Sognefjorden at locations some kilometers away from where the crossings first were done. In addition, we leveled a new line splitting the big loop in two. Finally, we remeasured the line marked in blue in the right panel of Figure [5.3.](#page-33-1)

In the right panel of Figure [5.3,](#page-33-1) the loop misclosure of 10.5 cm is now strongly reduced. On the other hand, the situation has become more complicated. The second biggest loop in southwest has a misclosure qualifying for a 17th place in Table [5.2.](#page-30-0) More serious is the extreme value of 6 cm for the small eastern loop crossing Sognefjorden three times. The enlarged map in Figure [5.4](#page-34-0) shows more details. Two of the fjord crossings in the north, Vangsnes-Eitorn and Hella-Dragsvik, are both measured two times. Vangsnes-Eitorn in 1963 and 2011, and Hella-Dragsvik in 1962 and 2004. Based on a separate outlier test, we rejected the 1963-crossing for Vangsnes-Eitorn, and it is not included in the loop misclosure of 6 cm. We were not able to identify any more erroneous line or fjord crossing. Hence, we used them all, hoping that the average is closer to the truth.

Figure 5.3: Left: The shaded loop has a misclosure of 10.5 cm and is 599 km long. Right: Loop misclosures across Sognefjorden after including new leveling data and new fjord crossing observations. Figure [5.4](#page-34-0) shows more details for the loop inside the black rectangle.

Figure 5.4: Overview of the loop with the largest loop misclosure. The crossings Vangsnes-Eithorn and Hella-Dragsvik are measured two times and the crossing Liktvoran-Kvamsøy is measured one time.

Also an area north of Sognefjorden was investigated in the period 2009 to 2011. In order to identify errors, we controlled several fjord crossings by remeasuring them and measured new leveling lines that split bigger loops.

As shown in Figure [5.5,](#page-35-1) we have split the big green loop into three separate ones. The result is somewhat difficult to interpret. The error seems to be located in the northern loop where there is a fjord crossing involved. However, also the two other loops have high misclosures. Note that there is no loop misclosure indicated for the loop below the big green one. The reason is that the western line was rejected in the 2008 adjustment, but it is still marked on the map. In the 2012 adjustment, the eastern line was releveled and rejection is not obvious anymore. Instead, we have rejected the old eastern line from 1936. From a statistical point of view, multippel t test does not identify the western line as an outlier, and we kept the line even if the misclosure is large. In our search for errors, we remeasured the two fjord crossings with the result shown in Table [5.4.](#page-35-2)

The 2012 adjustment followed the procedure outlined for the 2008 adjustment: First, final geopotentials on the nodal points were calculated, and then new heights at all points on lines connected to nodal points. The weighting strategy was the same and the downweighting of fjord crossings has a significant effect in this area, where we have many of them.

Figure 5.5: Loops north of Sognefjorden before (left) and after (right) the new measurements. High loop miclosures appear in new loops.

Railway lines and second order lines in the four western counties had to be recalculated based on the 2012 adjustment. We followed the same procedure as described in Section [5.4.](#page-31-1)

5.7 Can we trust the heights around Sognefjorden?

From Figure [5.6](#page-36-0) it is clear that the fjord crossings closest to the mouth of Sognefjorden have large effect on the estimated heights. To find support for keeping the western fjord crossing, we conducted an independent test where we made use of GNSS and a gravimetric geoid model. NKG released a new quasigeoid model for the Nordic countries in 2016, the NKG2015qgeoid (Ågren et al., [2016\)](#page-53-2). Trying to fit this model to our GNSS/leveling points by least squares collocation, would have revealed an error of 10 cm in the leveling network as a systematic shift in the signals over the fjord. We cannot see anything of this at the western crossing. The average correction to the GNSS/leveling point is 11 and 13 mm for the northern and southern side, respectively - using eight points located between the fjord crossing on the northern side and five on the southern. At the eastern crossing, we have not as many GNSS/leveling points between the locations of the crossings. Nevertheless, doing the same test here indicates a systematic shift of 33 mm when we use the six closest points on the northern side and the three on the southern (see Figure [5.7\)](#page-37-0).

At the time of calculation, NKG2016qgeoid did not exist and we could not perform the

Fjord	Year	Height difference (m)	Distance (m)
Dalsfjorden	1987	-18.781	1412
Dalsfjorden	2008	-18.792	1412
Førdefjorden	1987	-0.243	1217
Førdefjorden	2008	-0.241	1217

Table 5.4: Remeasured fjord crossings.
test described above. However, other models existed and gave approximately the same result. This indicated that keeping the western fjord crossing was a correct choice. At the eastern crossing, the fit is not so good, which is understandable keeping in mind the loop misclosure of 6 cm, which is large for a small loop and the worst for the entire Norwegian leveling network.

Figure 5.6: The difference between the preliminary 2008 adjustment and the final 2012 adjustment for the western part of Norway.

Figure 5.7: GNSS/leveling points on both sides of Sognefjorden. The colors indicate the sum of signal and trend when constraining the gravimetric geoid model NKG2015qgeoid to the points by least squares collocation. Green < 3.3 cm, light blue < 7.3 cm, and dark blue < 10.3 cm.

Chapter 6

The heights in the passive GNSS network (Landsnettet)

The previous chapters describe the implementation of new heights in the leveling network as the first realization of the new height system NN2000. The majority of the points in the GNSS reference network, usually called Landsnettet, are not directly connected to the Norwegian leveling network, so the heights have to be determined by other means. We have used the ellipsoidal heights in combination with a fitted geoid model to obtain the NN2000 normal heights in the Landsnett points. Fitted geoid models are normally called height reference (HREF) models. The same HREF-model is used for surveying with real-time kinematic (RTK) positioning systems. The points in Landsnettet are used as fixed points in surveying and mapping projects and a number of other purposes.

In this chapter, we will describe the steps of the implementation of NN2000 in all Norwegian municipalities. Most of the work described in this chapter was done as a cooperation between the Geodetic Institute and the regional offices of the NMA, as shown in Figure [6.1.](#page-39-0) The work was financed through the Geovekst cooperation.

Geovekst

Geovekst is a national program for co-operation on establishing digital geographic data in Norway. The basic concept is pooling of money for jointly executed projects for establishing, improving and maintaining large-scale digital geographic data. The general Geovekst program includes the State Road Department, the Board of Electricity Companies, the Norwegian Association of Local Authorities, Norwegian Mapping Authority, the Telecommunication Department, and the Ministry of Agriculture. Other services may participate in the program in specific regions. The Norwegian Mapping Authority undertakes the coordinating role both on national and regional level. The practical execution is organized as individual projects through which digital data are established and administrated in specific, limited geographic areas. The projects are based on an agreed set of standard rules and manuals, which facilitate the exchange and sharing of data across administrative boundaries.

Figure 6.1: Workflow for the implementation of NN2000.

6.1 The ellipsoidal heights

The existing passive GNSS network was established in the 1993-2008 period. The network consists of 12.000 control points with coordinates in EUREF89, the Norwegian realization of ETRS (European Terrestrial Reference System). The core part of the network (100 points) is measured by three-days GPS campaigns and densified by a network of GPS baselines with observation time from 1 to 4 hours. Initially, our focus was on the horizontal components. Later, it became clear that the quality of the ellipsoidal heights in EUREF89 was insufficient for serving as GNSS/leveling points in a fitted geoid model. Finding a suitable strategy for improving the ellipsoidal heights was subject to long internal discussions. The conclusion was to calculate the ellipsoidal heights in the new reference frame, IGS05N, using the campaign measurement as fixed points, i.e., these points were given infinitely large weights. The ellipsoidal heights were then transformed to EUREF89 by equations described in Appendix [A.](#page-54-0) The selected method for improving the ellipsoidal heights is described in the following paragraphs.

6.1.1 GNSS campaign measurements

From 2009, GNSS campaigns were conducted in order to evaluate the quality of EUREF89. The new GNSS coordinates were calculated in the reference frame IGS05N, epoch 2009.58, and based on stations in the Continuously Operating Reference Stations (CORS) network of Norway. The calculations were done in the Bernese GNSS Software. From 2010, the purpose of these campaigns changed. From then on, the GNSS campaigns were carried out on selected points in the control point network as a basis for updating the ellipsoidal heights. The idea was to realize a new reference frame in IGS05N with an associated transformation to EUREF89. This transformation is defined in Appendix [A.](#page-54-0) Another motivation for the GNSS campaigns was to densify the network of GNSS/leveling points. This was achieved by GNSS measurement on leveling benchmarks.

The GNSS campaign points were selected based on these criteria:

- Density about 30-40 km
- Bedrock or other stable foundation
- Good GNSS conditions
- Leveled height in NN2000

6.1.2 GNSS baselines and CPOS measurements

A subset of points were remeasured with GNSS in order to strengthen the original baseline network from 1993-2008. The horizontal position and the ellipsoidal height of the points were determined by observing baselines with static GNSS as well as the CPOS RTK network service [\(Ouassou](#page-52-0) et al., [2015\)](#page-52-0). At each point we wanted to strengthen, two baselines were observed over 2-4 hours, depending on conditions. One CPOS measurement was taken when setting up the equipment for observing the GNSS baselines, and a second was taken when taking it down, regardless of the distance from the CORS. One CPOS measurement was a combination of at least three registrations with separate initialization of the CPOS receiver. Note that a dedicated CPOS service, was set up in the reference frame IGS05N and the observed coordinates were consequently determined in IGS05.

Guidelines for the GNSS campaigns are given by [Kartverket](#page-51-0) [\(2020\)](#page-51-0). For example, the surveyors were required to test their CPOS equipment at the NMA head office in Hønefoss. The test was done by measuring a well-defined control point and comparing the measured height to the given height.

GNSS baseline measurement is time consuming, especially in fjord and mountain areas. To speed up the work, the CPOS service was tested as an alternative to GNSS baselines in 2012. A distance limit of 15 km from the nearest CORS was set for the CPOS measurements to keep up with accuracy requirements. For measurements further away from CORS, battery-powered temporary reference stations were used. The temporary stations were connected to the CORS network by the mobile phone network. The stations were set up over the campaign points discussed in Section [6.1.1.](#page-39-1) The surveyor visited each point three times. The time separation was set to at least six hours, and spread over two days. As it turned out, the logistics of this method were rather challenging. Measurements were going on simultaneously at several locations, and there was a shortage of available temporary reference-station kits. Poor mobile network coverage proved to be a problem in many areas. In addition, there was a risk of antenna height errors at the temporary reference stations. As a consequence, this approach was in use only in 2012.

6.1.3 Weighting of the campaign observations

As described in Section [6.1.1](#page-39-1) and Section [6.1.2](#page-40-0) the observations were of three types:

1. GNSS campaigns over five days, calculated in the Bernese GNSS Software.

- 2. Baselines with a length of 1 to 10 km observed over 1 to 4 hours, processed with various software.
- 3. CPOS measurements.

The individual observation weight as well as the relative weight between the different types of observations will influence the result. The weight is given as $1/\sigma^2$, where σ is the standard error of the observation.

For the baselines we found the standard error of the derived height difference by using the formula

$$
\sigma_{dh} = \sqrt{k_1^2 + k_2 d^2 + k_3 dh^2},\tag{6.1}
$$

where $k_1 = 0.01$ m, $k_2 = 0.001$, and $k_3 = 0.025$. d is the distance and dh the height difference, both in kilometers. We have found the three k-parameters in this formula by using the variance component estimation method described by [Mathisen](#page-51-1) [\(1977\)](#page-51-1). The third part in the formula is the most interesting one, penalizing vectors with larger height differences.

For the CPOS measurements, we used the standard error given by the CPOS system to calculate the weight. As mentioned in Section [6.1.2,](#page-40-0) a measurement consisted of at least three registrations. In the adjustment we used the mean value and the mean standard error of these three registrations to represent one measurement. The CPOS system provides standard errors of each component of the coordinate given in a geocentric Cartesian coordinate system. In addition, the correlations between each component are given. Following the law of error propagation, the standard error of a mean ellipsoidal height observation was calculated from the standard errors of each coordinate component. At each point we obtained two such mean observations, one when setting up the antenna for base line measurement and the second one when taking it down.

For the campaign measurements we could not use the standard error given by the Bernese GNSS Software directly. Those estimates were too optimistic. Two possible weighting strategies were discussed:

- 1. Use fixed campaign points in the adjustment, i.e., give them infinite weight.
- 2. Calculate weights based on variance component estimation.

The accuracy of the campaign coordinates was considered superior to the baselines. Thus, for practical reasons, the first strategy was selected.

For the last two observation types, the baselines and the CPOS measurements, we performed a simple variance component estimation procedure to make sure that the relative weight between them was correct. This typically gave standard errors of the unit weight close to one, for both observation types. If not, we scaled the two observation types relative to each other in order to obtain a value closer to one.

6.1.4 The calculation

We used IGS05N as reference frame. Since the ellipsoidal heights of the campaign points are fixed in the adjustment, we have two types of observations only:

- Height differences from GNSS baselines
- Observed ellipsoidal heights in IGS05N from CPOS

To avoid possible systematic errors in CPOS, we typically estimated an unknown bias for each day.

Usually the network covered a county or a part of a county, including some thousand observations and some thousand unknown heights. Before the final LSA, we tested all observations for outliers. The rejection criterion was set to three (see Section [5.1.3\)](#page-26-0) and rejected observations were flagged in the observation database as outliers. Another important test was the calculation of external reliability (the effect of the undetectable outliers on the estimated parameters) and the assignment of height classes. The requirements for the different classes are listed in Table [6.1.](#page-42-0)

The Norwegian guidelines for GNSS networks (Statens [kartverk,](#page-52-1) [2009a\)](#page-52-1), define requirements for the relative reliability of the height difference between two neighbor points. The relative reliability (Δ) is obtained by combining the numbers in Table [6.1](#page-42-0) with Equation [\(6.2\)](#page-42-1).

$$
\Delta = \sqrt{\mathstrut p^2 + 2\frac{k^2}{l^2}}
$$
\n(6.2)

Here l is the slope distance in kilometers between two points, and p and k are parameters given in Table [6.1.](#page-42-0) According to the Norwegian guidelines, the points in the GNSS reference network should have a relative reliability fulfilling the same requirement as stated for height class A. The guidelines say nothing about the other two criteria, i.e., the standard error and the absolute reliability. Those are internal requirements used by the NMA. This means that all points assigned to height class A, fulfill the Norwegian guidelines. It turns out that also points assigned to other classes than A, fulfill the Norwegian requirements. In total, 81.6% of the points in the GNSS reference network, fulfill the Norwegian guidelines for ellipsoidal heights, as shown in Table [6.2.](#page-43-0)

Defining quality criteria for a geodetic reference network is not straightforward. Depending on perspective and use, different criteria are preferred. We believe that the existing Norwegian regulations alone are not sufficient. For some purposes, the absolute reliability (the difference from a reference value) is a more useful quality indicator, e.g., when controlling the CPOS equipment. As a rule of thumb, points that belong to class A and B are qualified for most surveying and mapping purposes dealing with ellipsoidal heights, points in class C may be poor, and proints in height class F should be avoided if possible.

Table 6.1: Requirements for standard error and external reliability (relative and absolute) for the different ellipsoidal height classes.

		C	
Standard error ≤ 6 mm ≤ 8 mm ≤ 10 mm > 10 mm			
Relative reliability	$p=6$ ppm, $p=6$ ppm, $p=6$ ppm, $k=6$ mm $k=10$ mm $k=15$ mm		
Absolute reliability		$\langle 8 \text{ mm } \rangle$ $\langle 12 \text{ mm } \rangle$ $\langle 15 \text{ mm } \rangle$ $> 15 \text{ mm}$	

	A	B	$\mathbf C$	$\mathbf F$	Sum
Number of points	5761	4598	922	324	11605
Number of points in percent of all	49.6	39.6	7.9	2.8	100
Number of points fulfilling Norwegian requirements	5761	3180	416	115	9472
Number of points fulfilling Norwegian requirements in percent of the total	49.6	27.4	3.6	1(0)	81.6

Table 6.2: Statistics of different ellipsoidal height classes.

Both IGS05N and EUREF89 ellipsoidal heights are stored in the database. This means that we have realized the ellipsoidal heights of our GNSS reference network in two different reference frames that are related by a mathematical transformation. We also stored all three quality measures for all individual points and the assigned height class as an overall quality indicator.

6.2 Final NN2000 heights

The final NN2000 heights for our GNSS reference network were calculated by transforming the height components of the GNSS-observations to normal height differences by using the latest updated HREF model. The CPOS-observed ellipsoidal heights were transformed to normal heights using the same HREF model. An alternative approach could have been to transform ellipsoidal heights directly to normal heights in NN2000 using the HREF model. However, this method would degrade the leveled heights, and was therefore not used.

6.2.1 The gravimetric geoid model, GNSS/leveling points, and HREF

A main challenge of this procedure was that we needed a high-quality geoid model to obtain NN2000 heights on points in the GNSS reference network that are not leveled. To obtain the desired quality, gravimetric geoid models must be constrained to GNSS/leveling points, i.e., points that are leveled and with accurate and reliable ellipsoidal height. Such models are called height reference models or just HREF models. In Norway, the difference between a pure gravimetric model and the geoid heights derived through the GNSS/leveling points is more than 15 cm in the worst cases, even if we solve for a shift or a bias between them. The average difference is 3-4 cm in terms of RMS depending on the models.

The first challenge was to find a gravimetric geoid model on which we could base the

HREF model. The most recent geoid model from NKG at the time, the NKG2004 model, was out of date. Several Norwegian geoid models calculated by the NMA were tested, and the gravimetric geoid model NMA2013v30 was selected. Except for three pilot areas (Kristiansand area, Hamar-Lillehammer region, and Trondheim), we have used this model for the implementation of NN2000 in all Norwegian counties. In the pilot areas we used the NKG2004 model, the presumably best model at the time.

Another challenge was to establish a sufficiently dense network of GNSS/leveling points. The three major issues were:

- The extent and density of the precise leveling network.
- The distribution and the total number of GNSS/leveling points.
- The quality of the ellipsoidal heights in the GNSS/leveling points.

They were addressed as follows:

- 1. The leveling network was extended and densified such that the distance to the nearest GNSS/leveling point was less than 15 km in populated areas and along public roads.
- 2. GNSS campaigns were accomplished in existing or new GNSS/leveling points with a distance of approximately 15 km between them. If the horizon around the leveled benchmark was not good, an eccentric set-up was considered.
- 3. All other GNSS/leveling points were remeasured with CPOS and two baselines to neighboring points in the GNSS reference network as described in Section [6.1.2.](#page-40-0)

By the end of the project in 2018, these actions had resulted in 3299 GNSS/leveling points. These points were then used to fit the gravimetric geoid model NMA2013v30 to obtain HREF models. We have calculated all HREF models by least squares collocation. When making the models, we predicted values at all points in a grid with spacing $0.02°$ in the north-south direction and 0.04◦ in the east-west direction. This corresponds to \sim 2.2 km for both directions. Two types of HREF models were calculated, one referring to ellipsoidal heights in EUREF89 and another one to IGS05N. The complete names of the models are HREF2018a NN2000 Euref89 and HREF2018a NN2000 IGS05N. Note that before we reached the final model, a number of intermediate models were calculated and used when calculating heights in the municipalities. These intermediate models were successively updated regionally before the height calculation and make up the final one for this region. This means that for an updated region the intermediate model does not differ from the final one.

6.2.2 Heights in island communities

The leveling network does not cover the many islands along the coast of Norway, where the distance to the nearest GNSS/leveling point may by far exceed the recommended maximum distance of 15 km. The consequence is that the HREF model turns into a pure gravimetric geoid model. Our experience suggests that there is a risk of systematic errors in the geoid model in this type of landscape. Therefore, we have to determine the NN2000 height by other means.

Municipality	Island	Distance	Remark
Kvitsøy	Kvitsøy	$12 \;{\rm km}$ $+12$ km	Relative tide gauge observations only Connected both to Randaberg in south and to Arsvågen in north
Utsira	Kvitsøy	19 km	Relative tide gauge observations only
Solund	Sula.	$5 \;{\rm km}$	Relative tide gauge observations $+$ normal fjord crossing
Smøla	Smøla	6 km	Relative tide gauge observations $+$ normal fjord crossing
Hasvik	Sørøya	25 km	Relative tide gauge observations only

Table 6.3: Fjord crossings based on local tide gauge observations.

One option is normal fjord crossings using vertical angle measurements, but over distances longer than 5 km, the uncertainty will be higher than accepted. Another and more reliable approach, is to use temporary tide gauges to transfer the NN2000 height to the island. We connected both tide gauges to benchmarks by leveling, i.e., to a leveled benchmark on the mainland and to a point in the GNSS reference network on the island. The required length of the tide gauge record depends on distance, weather, oceanographic aspects, and the requested accuracy. For short distances and good conditions, the standard error converges to less than 1 cm after just a few hours. For longer distances, days are required to reach the same result. Furthermore, seasonal differences complicate the picture. The crucial factor, is that the water level on both sides of the fjord on average coincides with the same potential surface. If this condition is not fulfilled, the accuracy will degrade accordingly. The tide gauge method is used for the five crossings listed in Table [6.3.](#page-45-0)

We intended to use the same type of relative tide gauge measurements to several more island communities and municipalities in Møre og Romsdal, Nordland, and Finnmark, but there was no time to do this. Instead, a special combination of GNSS and HREF was used for the remaining islands. As already mentioned above, the underlying gravimetric model in the HREF-model has an increasing influence when moving away from the GNSS/leveling points, especially towards the outskirts of the model where extrapolation rather than interpolation determines the HREF values. Recently, the NKG2015qgeoid was released, which quality is expected to surpass the NMA2013v30 goeid on which the HREF model was based. For the island communities, the geoid heights from the NKG2015qgeoid were used to determine NN2000 heights on the island and later update the HREF model. The procedure followed five steps:

- 1. The NKG2015qgeoid-model was fitted to GNSS/leveling points on the mainland.
- 2. For one or more high quality Landsnett-points on the island, the NN2000 normal height was computed by subtracting the geoid height derived from the fitted NKG2015qgeoid model from the ellipsoidal height.
- 3. The island-points, were then added to the HREF model as new GNSS/leveling

Table 6.4: The calculation of NN2000 at central benchmarks in island municipalities. The second column lists the high-quality benchmarks on the islands, the third column their ellipsoidal height, the fourth column the raw value of the gravimetric model, the fifth column the predicted signal using least squares collocation and GNSS/leveling points on the main land as observations, and the sixth column the final NN2000 height (column 3 minus the sum of column 4 and 5).

Municipality	benchmark Central	Ellipsoidal $\binom{m}{k}$ height	height 5q 201 Geoid NKG:	correction Predicted	NN2000 height Final	Remark
Midsund	C26T0329	49.317	44.62	-0.091	4.788	
	C ₂₆ T ₀₁₆₆ C26T0327	46.135 56.473	44.73 44.791	-0.085 -0.103	1.490 11.785	
Haram	C27T0533	67.122	44.868	-0.058	22.312	Only for Haramsøyene
Meløy	J14T0202	36.234	33.646	0.000	2.588	
Vega	H17T0092	58.309	37.115	-0.047	21.241	
Træna	H15T0025	81.423	37.317	-0.038	44.144	
Lurøy	I15T0131	43.39	35.776	-0.068	7.682	Only for Lovund and Solvær
Radøy	I15T0139	65.661	34.777	-0.038	30.922	Only for the islands
Karlsøy	P04T0099 P04T0100	33.322 35.158	29.881 30.439	-0.011 -0.008	3.452 4.727	For Vannøya
Skjervøy	R04T0094	62.254	29.324	-0.023	32.953	For Arnøya

points, and an updated HREF model based on the NMA2013v30 geoid was calculated.

- 4. The normal heights in NN2000 of the remaining Landsnett points on the island were calculated based on the updated HREF model.
- 5. If there were any leveling lines on the island, one of the selected points in step two should be connected and the NN2000 heights in the line calculated from this.

This procedure was used for the island communities listed in Table [6.4.](#page-46-0)

6.3 The transformation between NN1954 and NN2000

The transformation of geographical data and map databases from NN1954 to NN2000 was the final task in the long process of implementing the new height system. The transformation model is based on a set of common points with reliable heights in both height systems.

The benchmarks in the leveling network are obvious candidates as common points. Away from the leveling lines, the points in the GNSS network are equally good candidates as they are used as reference points for a number of purposes including airborne light detection and ranging (LiDAR) measurements and aerial mapping.

The differences between the height systems NN2000 and NN1954 show an irregular pattern, see Figure [6.2.](#page-48-0) This means that a mathematical transformation would be inaccurate. Instead, we have made a grid model describing the differences. This transformation model is similar to the HREF models in terms of format and idea.

The model is based on least squares collocation. In addition to a parametric model of the differences, we also estimated signals describing the irregularities. In sum, the parametric part and the signals describe the difference between the two height systems.

The transformation model was updated regionally. The selection of common points was done in two stages:

- 1. Nodal points in the leveling network and some points in the lines connecting them. The model was then tested on the heights of points in the GNSS reference network in the area, typically a county.
- 2. Additional common points were added in areas where the test revealed systematic differences.

This procedure resulted in one or two new models every year, named by the year and a letter (a, b, c and so on). The final model was named NNTrans2018a. The model contains predicted values in a grid with spacing 0.025◦ in the north-south direction and $0.05°$ in the east-west direction, corresponding to around 2.8 km for both directions. We converted the grid file into a binary file, using a file format often referred to as the KMS format. Note that the file format is exactly the same as for the HREF models, only the spacing is different.

In least squares collocation, the covariance function plays a central role, as it tells the system about the correlation between the signals and their variance. It thus controls the smoothness of the final model. Especially important is the relationship between the variance of the observations, the common points, and the variance of the signals. We have run the collocation in two steps. In both steps, we used a polynomial surface of third degree as the parametric part and an exponential function to describe the signal's covariance (often this function is referred to as a Gauss-Markov process of first degree). We found the covariance by using the following model [\(Moritz,](#page-51-2) [1980\)](#page-51-2):

$$
C_{ss} = \sigma^2 e^{-\beta d}.\tag{6.3}
$$

In the first step, we set σ to 26 mm and β to ln(2)/35, i.e., the covariance reaches its half value after 35 km. The standard error of the common points, the observations, was set to 14 mm. In the second step, we reduced σ to 10 mm, β to $\ln(2)/5$, and the standard error of the observation to 10 mm, the same as for σ .

The parameters above result from many experiments. They seem to work reasonably well. Establishing such transformations is in many ways not an exact science. On one side, we aim at avoiding points that have erroneous heights in the old system, while on the other side, we might be forced to use them as common points when they have been reference points for the geographical data we will transform to the new height system.

As a practical solution, the Geodetic Institute provided the regional offices of the NMA with suggestions to which common points to use, as well as information on how well the

Figure 6.2: The difference between NN1954 and NN2000, the transformation model NNTrans.

remaining points fitted into the transformation. Normally the deviations were less than 3 cm. In Sogn og Fjordane, where the terrain is rough and the old heights inaccurate, the difference reached 5 cm in some points. Points in remote areas with large deviations were discarded, as surveyors most likely would never use them for precise height determination in the future.

When we had agreed upon what common points to use, the Geodetic Institute sent over the final transformation model, and the regional offices performed the transformation of the map databases. The map databases are shared among the partners in the Geovekst program, and thus, the municipalities, the road administration, and all other partners obtained updated map databases in NN2000.

Chapter 7

Key parameters of NN2000 and recommendations for further reading

The vertical reference frame NN2000, realized for Norway and implemented in all municipalities in the period 2013 to 2018, has the following characteristics:

For further reading, we recommend [Gerlach](#page-50-0) et al. [\(2013\)](#page-50-0); Harsson & [Pettersen](#page-50-1) [\(2014\)](#page-50-1) and [Revhaug](#page-52-2) [\(2019\)](#page-52-2).

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Appendix A

Transformation from IGS05N to EUREF89

The transformations from IGS05N to EUREF89 are similar to the transformations between ITRF and national realizations of ETRF, as outlined by [Nørbech](#page-52-3) et al. [\(2008\)](#page-52-3) and Häkli et al. [\(2016\)](#page-50-2). This procedure includes two steps. First, coordinates in IGS05N at epoch 2009.58 are transformed to IGS05N at epoch 1995.0 by Equation [\(A.1\)](#page-54-1). Then, Equation [\(A.3\)](#page-54-2) provides the transformation to EUREF89.

Transformation from reference epoch 2009.58 to 1995.0

Intraplate deformations in Fennoscandia between 1995.0 and 2009.58 are corrected by applying the NKG REF03vel velocity model. The horizontal part of this model originates from the glacial isostatic adjustment model by [Milne](#page-51-3) et al. [\(2001\)](#page-51-3) and the vertical part is based on NKG2005LU (Agren $&$ [Svensson,](#page-50-3) [2007\)](#page-50-3).

$$
\begin{bmatrix} x \\ y \\ y \\ \end{bmatrix}_{\substack{IGS05N \\ 1995.0}} = \begin{bmatrix} x \\ y \\ y \\ 10805N \\ 2009.58 \end{bmatrix} + (1995.0 - 2009.58) \cdot R \cdot \begin{bmatrix} \dot{n} \\ \dot{e} \\ y \\ \end{bmatrix}_{\text{NKG03}} \tag{A.1}
$$

The rotation matrix R transforms the velocities from a topocentric coordinate system $(n,$ \dot{e}, \dot{u} to a geocentric Cartesian coordinate system $(\dot{x}, \dot{y}, \dot{z})$. The rotation matrix is defined in, e.g., [Torge](#page-52-4) [\(2001,](#page-52-4) Equation 2.28):

$$
R = \begin{bmatrix} -\sin\varphi\cos\lambda & -\sin\lambda & \cos\varphi\cos\lambda \\ \frac{\sin\varphi\sin\lambda}{\cos\varphi} & \cos\lambda & \cos\varphi\sin\lambda \\ \cos\varphi & 0 & \sin\varphi \end{bmatrix} (A.2)
$$

Transformation from IGS05N to EUREF89 at epoch 1995.0

The 7-parameter Helmert transformation includes the effects of rigid plate motion and differences in reference frame realizations.

$$
\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\text{EUREFS9}} = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} + (1+D) \cdot \begin{bmatrix} 1 & R_z & -R_y \\ -R_z & 1 & R_x \\ R_y & -R_x & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\begin{subarray}{c} \text{IGS05N} \\ \text{IGS05N} \end{subarray}} \tag{A.3}
$$

The transformation parameters below were calculated by the Norwegian Mapping Authority using 46 points with coordinates in both IGS05N and EUREF89 given at the reference epoch 1995.0 (see Table [A.1\)](#page-56-0).

$$
T_x = -9.50 \cdot 10^{-2} \text{ m}
$$

\n
$$
T_y = 1.39 \cdot 10^{-2} \text{ m}
$$

\n
$$
T_z = -7.48 \cdot 10^{-2} \text{ m}
$$

\n
$$
D = 14.24 \cdot 10^{-9} \text{ rad}
$$

\n
$$
R_y = -6.8772088 \cdot 10^{-8} \text{ rad}
$$

\n
$$
R_z = 7.5243374 \cdot 10^{-8} \text{ rad}
$$

Table A.1: Coordinates of the 46 points used to calculate the transformation in Equation [\(A.3\)](#page-54-2) between IGS05N and EUREF89 at the reference epoch 1995.0. The EUREF89 coordinates differ slightly from the official values at that time since they are corrected for land uplift to the reference epoch 1995.0 using the model by [Danielsen](#page-50-4) [\(2001\)](#page-50-4). The official coordinates at that time referred to either 1994, 1995 or 1996.

	IGS05N			EUREF89		
Point	X	Y	Ζ	X	\overline{Y}	Ζ
AA03	3278077.8441	521844.1398	5428195.4668	3278078.2045	521843.8823	5428195.2420
AA04	3243165.3614	429952.9689	5457302.8749	3243165.7233	429952.7187	5457302.6656
AA05	3281097.3959	488819.1034	5429535.3086	3281097.7592	488818.8502	5429535.0969
AK05	3099618.3074	617374.9992	5521715.5940	3099618.6841	617374.7678	5521715.3852
BU01	3187312.2539	544755.3068	5479521.5761	3187312.6175	544755.0618	5479521.3555
BU03	3173492.7273	552661.1213	5486564.2360	3173493.0901	552660.8757	5486564.0195
BU ₀₄	3166703.4731	524374.7549	5493381.5346	3166703.8447	524374.5121	5493381.3274
BU ₀₅	3148909.1743	574088.8678	5498631.3924	3148909.5506	574088.6298	5498631.1872
BU07	3146682.2836	536793.5440	5503536.0791	3146682.6540	536793.3063	5503535.8683
BU ₀₉	3126628.8848	549195.3273	5513697.7571	3126629.2555	549195.0881	5513697.5482
BU11	3128277.3688	424193.4348	5524926.4100	3128277.7274	424193.1894	5524926.2007
F101	2010883.3560	871741.0253	5969789.2207	2010883.7627	871740.8691	5969789.0925
FI02	1977931.8254	922156.9368	5973108.8674	1977932.2431	922156.7899	5973108.7590
HE01	2988028.9400	655957.2604	5578669.2278	2988029.3209	655957.0307	5578669.0256
HE ₀₂	3108470.6451	661270.7202	5511756.3640	3108471.0299	661270.4800	5511756.1535
HE03	3069510.5511	652308.8352	5534424.7630	3069510.9293	652308.6041	5534424.5540
HE04	3062695.0338	599495.7977	5544011.4761	3062695.4124	599495.5552	5544011.2612
HE ₀₅	3048371.6604	624628.5707	5549217.3292	3048372.0420	624628.3411	5549217.1302
HE ₀₆	2988391.3932	583379.2328	5586113.2903	2988391.7720	583378.9981	5586113.0907
H ₀₀₁	3205325.4959	343742.4332	5485280.7047	3205325.8426	343742.1820	5485280.4958
HO04	3129230.4736	375091.3994	5526674.8220	3129230.8300	375091.1566	5526674.6203
H _{O06}	3116481.9949	$350935.5059\,$	5535349.3723	3116482.3516	350935.2610	5535349.1617
MR01	2919455.1049	392482.4747	5638243.2184	2919455.4745	392482.2476	5638243.0322
MR08	2882133.8982	413951.1790	5655885.5640	2882134.2598	413950.9559	5655885.3691
NO ₀₂	2382528.2053	657261.9907	5860248.8590	2382528.5936	657261.8027	5860248.7081
NO ₀₃	2277560.9620	655910.5926	5901596.7103	2277561.3496	655910.4154	5901596.5638
NO13	2444605.8765	598587.6939	5840988.1610	2444606.2668	598587.5011	5840988.0028
NO16	2327352.2146	664908.6588	5881668.6825	2327352.6074	664908.4755	5881668.5323
NO17	2336650.8528	626667.9973	5881804.4095	2336651.2441	626667.8157	5881804.2613
NT ₀₄	2807246.1386	541526.3414	5682404.2590	2807246.5154	541526.1202	5682404.0807
NT ₀₆	2701426.6558	551988.5594	5732131.0853	2701427.0309	551988.3494	5732130.9109
OP ₀₁	3122014.4117	589817.8019	5512228.5918	3122014.7866	589817.5633	5512228.3823
OP02	3030855.1196	557051.0868	5565813.4042	3030855.4976	557050.8561	5565813.2109
OP ₀₃	3016020.9547	423676.1925	5586866.1129	3016021.3108	423675.9519	5586865.9057
OP ₀₄	2974674.6012	401250.6875	5609877.4675	2974674.9594	401250.4535	5609877.2590
OP05	2954720.6252	479776.1959	5614303.6532	2954720.9988	479775.9674	5614303.4664
OP ₀₆	2944196.9228	420762.2545	5623991.2180	2944197.2933	420762.0206	5623991.0262
OP ₀₈	3059018.5516	500520.6950	5556588.3775	3059018.9178	500520.4591	5556588.1667
OP ₀₉	3037059.6446	471822.2467	5571395.0041	3037060.0043	471822.0062	5571394.7948
OP11	2983891.0598	501190.7183	5596423.3999	2983891.4328	501190.4840	5596423.2095
OP12	2983498.8276	449804.5737	5601016.1306	2983499.1879	449804.3394	5601015.9272
ST ₀₆	2817277.3753	454318.6003	5685095.5226	2817277.7515	454318.3813	5685095.3501
ST ₀₈	2727005.9733	505994.0458	5724330.6461	2727006.3536	505993.8335	5724330.4655
TE02	3230138.2882	484265.1408	5460332.3212	3230138.6543	484264.8933	5460332.1085
TE04	3189685.4170	403407.9670	5491275.1352	3189685.7666	403407.7159	5491274.9161
TR02	2102021.9747	719850.9158	5958615.1455	2102022.3738	719850.7450	5958615.0063

Table A.2: Residuals (in meters) of the transformation from IGS05N to EUREF89 at epoch 1995.0. The residuals are given both in a Cartesian coordinate system (dX, dY, dZ) and transformed to a topocentric coordinate system (dN, dE, dU) . The residuals are graphically illustrated in Figure [A.1](#page-58-0) and [A.2.](#page-59-0)

Point	dX	dY	dZ	dN	dE	dU
AA03	0.004	0.005	0.004	-0.002	0.004	0.006
AA04	-0.003	-0.001	-0.009	-0.002	-0.001	-0.009
AA05	-0.001	-0.000	-0.009	-0.003	-0.000	-0.008
AK05	-0.001	-0.007	0.003	0.004	-0.007	0.001
BU01	0.005	-0.001	0.007	-0.000	-0.002	0.009
BU03	0.006	0.001	0.004	-0.003	-0.000	0.007
BU04	-0.004	-0.002	-0.005	0.002	-0.001	-0.006
BU05	-0.005	-0.005	-0.005	0.003	-0.004	-0.007
BU07	-0.002	-0.005	0.001	0.003	-0.005	-0.001
BU09	-0.001	-0.002	0.000	0.001	-0.002	-0.000
BU11	0.003	0.002	0.000	-0.003	0.002	0.002
F101	0.003	0.001	0.004	-0.002	-0.000	0.005
FI02	-0.004	-0.005	-0.013	0.001	-0.003	-0.014
HE01	-0.000	-0.000	0.005	0.002	-0.000	0.004
HE02	-0.007	0.002	0.004	0.007	0.003	0.000
HE03	0.000	-0.005	0.005	0.003	-0.005	0.004
HE04	-0.004	0.006	0.011	0.008	0.007	0.009
HE05	-0.005	-0.005	-0.003	0.003	-0.004	-0.006
HE06	-0.003	0.004	0.002	0.003	0.004	0.000
HO01	0.007	0.001	-0.007	-0.009	0.000	-0.002
HO04	0.002	-0.001	-0.008	-0.005	-0.002	-0.006
HO06	-0.000	0.001	0.002	0.001	0.001	0.002
MR01	-0.006	-0.002	-0.007	0.002	-0.001	-0.009
MR08	0.005	-0.003	0.004	-0.002	-0.003	0.006
NO02	0.003	0.002	-0.001	-0.004	0.001	0.000
NO03	0.005	-0.001	0.002	-0.003	-0.002	0.004
NO13	-0.004	0.002	0.001	0.004	0.003	0.000
NO16	-0.000	0.002	0.002	0.001	0.002	0.002
NO17	-0.001	-0.001	-0.000	0.001	-0.001	-0.001
NT ₀₄	-0.000	0.003	-0.006	-0.003	0.003	-0.005
NT ₀₆	0.004	-0.001	-0.002	-0.004	-0.001	0.000
OP01	-0.002	-0.002	0.001	0.003	-0.002	0.000
OP ₀₂	-0.005	-0.004	-0.008	0.001	-0.003	-0.010
OP ₀₃	0.008	0.005	0.007	-0.004	0.004	0.010
OP ₀₄	0.005	0.001	0.011	0.000	0.001	0.012
OP ₀₅	-0.004	-0.002	-0.009	-0.000	-0.001	-0.010
OP ₀₆	-0.005	0.004	-0.003	0.002	0.004	-0.005
OP ₀₈	0.002	-0.001	0.007	0.002	-0.002	0.007
OP ₀₉	0.007	0.004	0.007	-0.003	0.003	0.010
OP11	-0.003	0.002	-0.007	-0.001	0.003	-0.008
OP12	0.006	0.002	0.005	-0.003	0.001	0.008
ST ₀₆	-0.006	-0.001	-0.013	-0.000	-0.000	-0.014
ST ₀₈	-0.005	-0.001	0.002	0.005	0.000	0.000
TE ₀₂	-0.003	-0.002	-0.004	0.001	-0.002	-0.005
TE04	0.009	0.003	0.005	-0.005	0.002	0.009
TR02	-0.000	0.007	0.008	0.001	0.006	0.008

Figure A.1: Residual vectors $(\sqrt{dN^2 + dE^2})$ of the transformation from IGS05N to EU-REF89 at epoch 1995.0.

Figure A.2: Height residuals (dU) of the transformation from IGS05N to EUREF89 at epoch 1995.0. See Figure [A.1](#page-58-0) for site-identifiers.

Appendix B

Results from the 2008 adjustment

B.1 Geopotential numbers from the Baltic Leveling Ring adjustment

Table B.1: Nodal points with geopotential numbers from the Baltic Leveling Ring adjustment.

Point ID	North $[m]$	East $[m]$	Geopotential [g.p.u.]
61237	7857857.230	1182283.550	66.519
2241402	7162736.160	746847.193	305.200
H24N0020	7026812.460	653151.320	546.538
H27N0064	6949443.500	657286.524	818.968
H ₂₉ N ₀₀ 35	6863891.570	667347.350	654.173
H35N0054	6644214.950	666963.340	130.710
H36N0031	6597562.300	655638.500	235.952
H36N0041	6588925.000	655809.600	150.140
H36N0058	6616243.800	661302.090	132.191
H37N0033	6558769.100	658858.200	143.847
H37N0080	6553334.370	630015.520	63.933
H38N0006	6535888.670	653642.050	142.720
I23N0006	7066548.440	663713.577	513.140
I24N0001	7062044.450	661220.622	426.958
I28N0003	6885336.010	669050.900	784.098
I30N0036	6830803.580	685466.034	568.950
I31N0024	6798606.940	705711.878	408.250
I31N0037	6773366.450	694120.235	303.070
I31N0073	6776970.910	698879.067	387.233
I31N0075	6787839.150	701750.209	404.652
I32N0007	6733052.350	688503.349	234.921
I32N0053	6764930.050	674549.412	435.797
I33N0033	6699806.880	697599.491	280.748

B.2 Geopotential numbers of nodes in the Norwegian leveling network

Table B.2: Geopotential numbers of nodes in the Norwegian leveling network.

B.3 Geopotential differences for Norwegian leveling lines

Table B.3: Geopotential differences for lines in the Norwegian leveling network.

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